Algorithm Analysis

$f(n), g(n)$: functions on n

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f = O(g) \\ \infty & f = \Omega(g) \\ c & f = \Theta(g) \\ \text{undefined} & \text{try something else} \end{cases}$$

Examples

$f = n^a, \quad g = n^b$

$f = \log(n), \quad g = n$

$f = 1, \quad g = \log(n)$

Algorithm Analysis

$y = \log_b x \iff x = b^y$

powers

$x^p \cdot x^r = x^{(p+r)}$

$x^p / x^r = x^{(p-r)}$

$(x^p)^r = x^{p \cdot r}$

$x^{p^r} = x^{(p^r)} \neq x^{p \cdot r}$

$a^{\log_b c} = c^{\log_b a}$

logarithms

$\log_b 1 = 0$

$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$

$\log_b(x/y) = \log_b(x) - \log_b(y)$

$\log_b(x^p) = p \cdot \log_b(x)$

$\log_b(x) = \dfrac{\log_a(x)}{\log_a(b)}$

Algorithm Analysis

if $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ is undefined $\left(\frac{0}{0}, \frac{\infty}{\infty}, \text{etc.}\right)$

then use L'Hôpital's rule.

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$$

Derivitives

$$\frac{d\, x^a}{dx} = a x^{a-1}$$

$\cancel{d}$ $\cancel{\log}$

$$\frac{d \ln(x)}{dx} = \frac{1}{x}$$

$$\frac{d\, a^x}{dx} = a^x \ln(a)$$

$$\frac{d \ln(x^a)}{dx} = \frac{d}{dx} a \ln(x) = \frac{a}{x}$$

$$\frac{d\, [\ln(x)]^a}{dx} = \frac{a[\ln(x)]^{a-1}}{x}$$

$$\frac{d\, x \ln(x)}{dx} = \ln(x) + 1$$

$$\frac{d\, x^a b^x}{dx} = a x^{a-1} b^x + x^a b^x \ln(b)$$

Algorithm Analysis

$a > 0$, $b > 0$

$1 = O(\log(n)^a)$

$\log(n)^a = O(n^b)$

$n^a = O(n^b)$ iff $b \geq a$

$n^a = O(b^n)$ iff $b > 1$

$a^n = O(b^n)$ iff $b \geq a$

$a^n = O(n^n)$

$a^n = O(n!)$

$n! = O(n^n)$

Big-Oh Cheat Sheet

You should add to this as new rankings are found.