Computational Theory

Reducibility

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Utah Tech University—Computing

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Undecidability

Reading: Sipser §5.1.

Theorem 5.1 $HALT_{\text{TM}}$

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Theorem 5.1: $HALT_{\text{TM}}$ is undecidable.

Proof Idea: Use a proof by contradiction. Assume $HALT_{\text{TM}}$ is decidable. Use $HALT_{\text{TM}}$'s decider to construct a decider for A_{TM}. By theorem 4.11, A_{TM} is undecidable. This is a contradiction. We will call this a reduction of A_{TM} to $HALT_{\text{TM}}$.

Theorem 5.1 $HALT_{\text{TM}}$

$HALT_{\text{TM}} = \{\langle M, w\rangle | M$ is a TM, w is a string, and M halts on $w\}$

Theorem 5.1: $HALT_{\text{TM}}$ is undecidable.

Proof: Assume $HALT_{\text{TM}}$ is decidable by R. Construct a decider for A_{TM}.

Let $S =$ "On input $\langle M, w\rangle$ an encoding of a TM and a string:

1. Run R on input $\langle M, w\rangle$.
2. If R rejects, *reject*.
3. If R accepts, simulate M on w until it halts.
4. If M accepted, *accept*; if M rejected, *reject*."

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4. If M accepted, *accept*; if M rejected, *reject*."

If M accepts w, S accepts. If M rejects w, S rejects. If M loops on w, S rejects. This is a decider for A_{TM}. This is the contradiction.

Theorem 5.2 E_{TM}

$E_{\mathsf{TM}} = \{\langle M \rangle | M$ is a TM and $L(M) = \emptyset\}$

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Theorem 5.2: E_{TM} is undecidable.

Proof: Assume E_{TM} is decidable by R. Construct a decider for A_{TM}.

Intermediate step.

Let $M_1 =$ "On input x:

1. if $x \neq w$, *reject*.
2. if $x = w$, run M on input w and *accept* if M does."

Note that $L(M_1) = \emptyset$ if M does not accept w, and $L(M_1) = \{w\}$ if M accepts w.

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Theorem 5.2: E_{TM} is undecidable.

Proof: Continued

Let $S =$ "On input $\langle M, w\rangle$ an encoding of a TM and a string:

1. Construct M_1 from M and w as described above.
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1. Construct M_1 from M and w as described above.
2. Run R on input $\langle M_1 \rangle$.
3. If R accepted, *reject*; if R rejected, *accept*."

If M accepts w, then the $L(M_1) \neq \emptyset$, so R will reject $\langle M_1 \rangle$, and S accepts. If M does not accept w, then the $L(M_1) = \emptyset$, so R will accept $\langle M_1 \rangle$, and S rejects. This is a decider for A_{TM}. Contradiction with theorem 4.11.

Mapping Reductions

Reading: Sipser §5.3.

Definition 5.17

A function $f : \Sigma^* \to \Sigma^*$ is a ***computable function*** if some Turing machine M, on every input w, halts with just $f(w)$ on its tape.

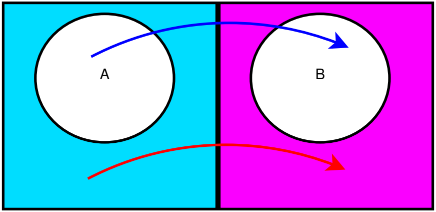
Definition 5.20

Language A is ***mapping reducible*** to language B, written $A \leq_m B$, if there is a computable function $f : \Sigma^* \to \Sigma^*$, where for every w,

$$w \in A \Leftrightarrow f(w) \in B.$$

The function f is called the ***mapping reduction*** of A to B or the ***reduction*** of A to B.

Definition 5.20



$$w \in A \Leftrightarrow f(w) \in B.$$

The function f is represented by the two arrows in the diagram above. The blue arrow indicates $w \in A \Rightarrow f(w) \in B$. The red arrow indicates $w \in \overline{A} \Rightarrow f(w) \in \overline{B}$.

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N decides A. Why?

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3. If M accepts $f(w)$, then *accept*. Otherwise *reject*."

N halts. Why? f is a computable function. M is a decider.

N decides A. Why? If $w \in A$, then $f(w) \in B$, and M accepts.
If $w \notin A$, then $f(w) \notin B$, and M rejects.

Corollary 5.23

If $A \leq_m B$ and A is undecidable, then B is undecidable.

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If $A \leq_m B$ and A is undecidable, then B is undecidable.

Proof: If B is decidable, then A is decidable. Since A is undecidable, B can't be decidable.

Using Reductions

- Our main technique for proving undecidability is to reduce from a known undecidable language to a suspected undecidedable language. This will become standardized with mapping reductions.
- Our main technique for proving decidability has been to provide a decider. We can now also use reduction from a suspected decidable language to a known decidedable language.

Reduction Proofs

Steps to a reduction proof.

- Select the language to be reduced from, A, and describe its instances.
- Select the language to be reduced to, B, and describe its instances.
- Provide the reduction function that maps *any* instance of A into *some* instance of B. Be sure to prove it is a computable function.
- Prove that $w \in A \Rightarrow f(w) \in B$.
- Prove that $w \in \overline{A} \Rightarrow f(w) \in \overline{B}$. Sometimes this is easier to prove by showing the equivalent $f(w) \in B \Rightarrow w \in A$.
- Conclude that $A \leq_m B$.

Reduction Proofs

Due to space considerations in these slides, we will not label each of these steps. But you should observe that they are all there. Assignments and exam questions will require you to use these steps, and label them.

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From theorem 4.11, A_{TM} is undecidable. From Corollary 5.23 If $A_{\mathsf{TM}} \leq_m HALT_{\mathsf{TM}}$, then $HALT_{\mathsf{TM}}$ is undecidable.

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Our task is to provide the reduction. That is, the computable function that mapping reduces A_{TM} to $HALT_{\text{TM}}$.

The Reduction of A_{TM} to $HALT_{\text{TM}}$

Let $F =$ "On input $\langle M, w\rangle$:

1. Let $w' = w$.
2. Construct the following machine M'.
   $M' =$ "On input x:
   1. Run M on input x.
   2. If M accepts, *accept*.
   3. If M rejects, *loop*."
3. Output $\langle M', w'\rangle$."

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Analysis: $\langle M, w \rangle \in A_{\text{TM}} \Leftrightarrow F(\langle M, w \rangle) = \langle M', w' \rangle \in HALT_{\text{TM}}$

- F halts. Note that F does not run M', it only computes it.

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From theorem 5.2, E_{TM} is undecidable. From Corollary 5.23 If $E_{\text{TM}} \leq_m EQ_{\text{TM}}$, then EQ_{TM} is undecidable.

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From theorem 5.2, E_{TM} is undecidable. From Corollary 5.23 If $E_{\text{TM}} \leq_m EQ_{\text{TM}}$, then EQ_{TM} is undecidable.

Our task is to provide the reduction. That is, the computable function that mapping reduces E_{TM} to EQ_{TM}.

The Reduction of E_{TM} to EQ_{TM}

Let $G =$ "On input $\langle M \rangle$:

1. Let $M_1 = M$.
2. Construct the following machine M_2.
   $M_2 =$ "On input x:
   1. *reject*."
3. Output $\langle M_1, M_2 \rangle$."

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Analysis: $\langle M \rangle \in E_{\text{TM}} \Leftrightarrow G(\langle M \rangle) = \langle M_1, M_2 \rangle \in EQ_{\text{TM}}$

- G halts.

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Analysis: $\langle M \rangle \in E_{\text{TM}} \Leftrightarrow G(\langle M \rangle) = \langle M_1, M_2 \rangle \in EQ_{\text{TM}}$

- G halts.
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- G halts.
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- If $L(M) \neq \emptyset$, $L(M_1) \neq L(M_2) = \emptyset$.

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Analysis: $\langle M, w \rangle \in A_{\text{TM}} \Leftrightarrow H(\langle M, w \rangle) = \langle M_1 \rangle \in \overline{E_{\text{TM}}}$

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- H halts.
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The Reduction of A_{TM} to $\overline{E_{\text{TM}}}$

Let $H =$ "On input $\langle M, w \rangle$:

1. Construct the following machine M_1.
   $M_1 =$ "On input x:
   1. If $x \neq w$, *reject*.
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N recognizes A.

Corollary 5.29

If $A \leq_m B$ and A is not Turing-recognizable, then B is not Turing-recognizable.

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If $A \leq_m B$ and A is not Turing-recognizable, then B is not Turing-recognizable.

Proof: If B is Turing-recognizable, then A is Turing-recognizable. Since A is not Turing-recognizable, B can't be Turing-recognizable.

Using Corollary 5.29

A typical use of Corollary 5.29 is to prove that a language, B, not Turing-recognizable.

- $\overline{A_{\textsf{TM}}}$ is not Turing-recognizable. (Corollary 4.23)
- $A \leq_m B \Leftrightarrow \overline{A} \leq_m \overline{B}$. By definition.
- Provide a mapping reduction $A_{\textsf{TM}} \leq_m \overline{B}$.
- This means $\overline{A_{\textsf{TM}}} \leq_m B$.
- By Corollary 5.29, B is not Turing-recognizable.

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We will use reductions from A_{TM} to $\overline{EQ_{\text{TM}}}$ and EQ_{TM}. These reductions will be paired with Corollary 5.29.

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Proof strategy: Providing a reduction from $A_{\textsf{TM}}$ to $\overline{EQ_{\textsf{TM}}}$. Deduce that $\overline{A_{\textsf{TM}}} \leq_m EQ_{\textsf{TM}}$. By corollary 4.23, $\overline{A_{\textsf{TM}}}$ is not Turing-recognizable. By corollary 5.29, $EQ_{\textsf{TM}}$ is not Turing-recognizable.

Our task is to provide the reduction. That is, the computable function that mapping reduces $A_{\textsf{TM}}$ to $\overline{EQ_{\textsf{TM}}}$.

The Reduction of A_{TM} to $\overline{EQ_{\mathsf{TM}}}$

Let $f =$ "On input $\langle M, w \rangle$:

1. Construct the following machine M_1.
   $M_1 =$ "On input x:
   1. *reject*."
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   $M_2 =$ "On input x:
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Analysis: $\langle M, w\rangle \in A_{\text{TM}} \Leftrightarrow f(\langle M, w\rangle) = \langle M_1, M_2\rangle \in \overline{EQ_{\text{TM}}}$

- f halts.

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Proof strategy: Provide a reduction from A_{TM} to EQ_{TM}. Deduce that $\overline{A_{\mathsf{TM}}} \leq_m \overline{EQ_{\mathsf{TM}}}$. By corollary 4.23, $\overline{A_{\mathsf{TM}}}$ is not Turing-recognizable. By corollary 5.29, $\overline{EQ_{\mathsf{TM}}}$ is not Turing-recognizable.

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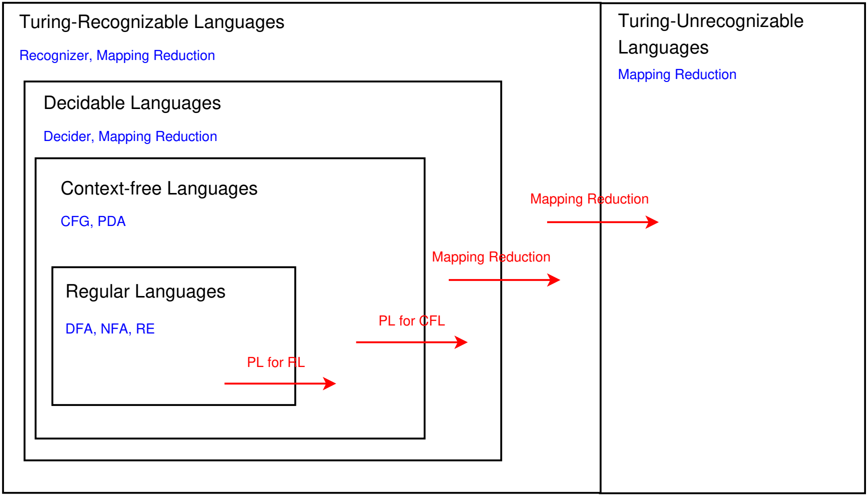
Summary


Turing-Recognizable Languages
Recognizer, Mapping Reduction
Decidable Languages
Decider, Mapping Reduction
Context-free Languages
CFG, PDA
Regular Languages
DFA, NFA, RE
PL for RL
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- **DEFINITION 5.20** mapping reducible.
- **THEOREM 5.22** If $A \leq_m B$ and B is decidable, then A is decidable.
- **COROLLARY 5.23** If $A \leq_m B$ and A is undecidable, then B is undecidable.
- **THEOREM 5.28** If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-recognizable.
- **COROLLARY 5.29** If $A \leq_m B$ and A is not Turing-recognizable, then B is not Turing-recognizable.

- **THEOREM 5.1** $HALT_{\text{TM}}$ is undecidable.
- **THEOREM 5.2** E_{TM} is undecidable.
- **THEOREM 5.3** $REGULAR_{\text{TM}}$ is undecidable.
- **THEOREM 5.4** EQ_{TM} is undecidable.
- **DEFINITION 5.17** computable function.
- **DEFINITION 5.20** mapping reducible.
- **THEOREM 5.22** If $A \leq_m B$ and B is decidable, then A is decidable.
- **COROLLARY 5.23** If $A \leq_m B$ and A is undecidable, then B is undecidable.
- **THEOREM 5.28** If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-recognizable.
- **COROLLARY 5.29** If $A \leq_m B$ and A is not Turing-recognizable, then B is not Turing-recognizable.
- **THEOREM 5.30** If EQ_{TM} is neither Turing-recognizable nor co-Turing-recognizable.